

Adding Stories to the Equation

Exploring the Narrative Potential of Late Medieval and Early Modern Reckoning Books

Reckoning books are an important but mostly overlooked source for the narration of knowledge. They grow especially important on the verge of the early modern period, where emerging European long-distance trade connections and the rise of the merchant class result in an unprecedented thirst for mathematical knowledge. Reckoning books now appear all over the continent, trying to tell the story of mathematics in short didactic examples. This article therefore aims to look at the reckoning books from a philological, narratological perspective and to grasp the narrative transfer of knowledge in the short form of the reckoning example. It is shown that these very brief examples are effective because they evoke so-called proto-narratives in just a few words and use narration as a potent tool of knowledge transfer.

1. A New Age of Numbers

As philologists, we generally tend to steer clear of numbers and mathematics – after all, we are seemingly working outside the realm of sheer measurability and mathematical facts. However, this means that we are missing out on a potential that has yet to be uncovered: Narration is of crucial importance for conveying pragmatic instructional knowledge, and instructional texts are often infused with storytelling.¹ Mathematical knowledge is no exception from this: This article therefore aims to look into the stories that are told when teaching mathematics.²

To do this, the study focuses on the 15th and 16th centuries, a period usually associated with transformation, upheaval and sweeping changes in almost every area of society – in short, a time where the world lingered on the brink of modernity. This also affected the practice of mathematics: The ability to calculate was becoming increasingly important because of expanding trade and the subsequent rise of a powerful mercantile elite. It was precisely these developments that depended on a skillset of more complex mathematical knowledge. Therefore, the transmission of mathematical knowledge in this period is an especially fruitful subject for examination. It is inextricably linked to the rise and widespread circulation of reckoning books written in the vernacular, which present, preserve, and pass on mathematical knowledge in a way that appeals to the general public.

To contextualize the popularity of mathematical pragmatic literature in the early modern period, a brief historical overview is provided: The “commercial revolution” of the 12th to 14th centuries (Denzel / Häberlein 2024, 191), which originated in Italy and spread throughout Europe, paved the way for a prospering long-distance trade: On one hand, continental Europe “served as a market for industrial and luxury goods from across the Mediterranean and the Orient, which were distributed to their respective hinterlands via Venice, Genoa, Barcelona, and other port cities. On the other hand, it supplied precious metals, raw materials, and manufactured goods [...] [transl. by the author]” (Denzel / Häberlein 2024, 191). With this prospering trade,³ the establishment of a fixed monetary system (Gärtner 2000, 14–15, 100) and the consequent integration of a big part of society in mercantile activities (Ewert et al. 2023, 495), cities expanded and a new social class emerged: the merchant class. Driven by the rise of the monetary system and the spread of market-based economic relations, this new elite gained self-confidence, challenging the established feudal system: As the economy expands, it creates connections and capital flows that transcended spheres of regional control (Leidinger 2008).

All these developments are dependent on the need for more complex mathematical skills. Kurt Vogel poignantly refers to this heightened diffusion of mathematical expertise as the “renaissance of mathematics” (Vogel 1954, 2) in the Occident. Mathematical skills now needed to be disseminated in the respective vernacular languages, and the knowledge was so sought-after that it was preserved in specialized reckoning books and taught in reckoning schools. The creation and the subjects of these schools can be traced back to the ‘*scuole d’abaco*’ (Caflisch 2018, 336–346) in northern Italy and the influence of Leonardo da Pisa’s (*c.1170; † after 1240) groundbreaking reckoning book, the *Liber abaci* (1202). His work combined fundamental arithmetic theory with practical, applicable examples, ready to be used for teaching, studying and practicing (Folkerts / Hughes 2016, 79–85). With his book, Leonardo – better known as Fibonacci – also brought another change to European mathematics:⁴ It established the use of Hindu-Arabic numerals (Folkerts / Hughes 2016, 208), the numerals taught and used throughout Europe to this day.⁵

The concept of teaching mathematical, mercantile knowledge and the use of the ‘new’ Hindu-Arabic Numerals also spread throughout the German-speaking regions: The profession of the reckoning master (the ‘*Rechenmeister*’) emerged, a craftsman leading reckoning schools as a business (Gärtner 2000, 14–15). Mathematical knowledge was recorded in a wide array of vernacular arithmetical treatises and reckoning books (‘*Rechenbücher*’). Intended recipients include merchants, aspiring traders and the young students of reckoning schools (Gärtner 2000, 234). The large number and widespread dissemination of these sources attest to their popularity and importance.⁶ Reckoning books mark a “turning point in the history of mathematics: on the one hand, in terms of their content and results, on the other, in terms of presentation, methods, and the application of mathematical knowledge [transl. by the author]” (Gärtner 2000, 12). They were tailored to practical use and taught the method of calculation with a new

numeral system, conveying an application-oriented, replicable “knowing how” (Krämer 2007, 43–45).

The strong practical focus of these books is reflected not only in their widespread dissemination, but also in their structure: A theoretical introduction to the basic arithmetic operations such as addition, subtraction, multiplication and division, both in integers and in fractions, is usually followed by an extensive collection of examples, often referred to as a *practica* (cf. Gärtner 2000; Vogel 1954). These examples merit closer analysis for several reasons: As in many other textual forms, such as legal discourse or moral didactic literature, instructional purposes are often inextricably intertwined with storytelling. In the reckoning books, this primarily takes place in the individual reckoning example, which serves as a kind of ‘case study’ to aid the transmission of mathematical expertise. At the core of each example lies a mathematical operation whose underlying processes have remained relatively stable, sometimes over centuries.⁷ On the contrary, the story that is woven around this mathematical core is variable and subject to a change determined by the circumstances of its time and its intended recipients. But even though reckoning books flourished throughout European vernaculars in the late Middle Ages and the early modern period, they have – in contrast to many other types of pragmatic texts – not yet received much attention, especially from a philological perspective.⁸ And whereas historians of mathematics have focused on the mathematical knowledge and the computational operations, this article is more interested in showing philological approaches to the ‘packaging’ of this expertise, namely by looking into the surrounding narratives in the reckoning examples.

2. Reckoning Examples – A Closer Look

Reckoning examples are an integral part of the reckoning book, as they effectively transmit the practical and replicable knowledge the texts are intending to impart. They convey a mathematical operation and enable the recipient to solve similar calculations on their own, thus taking on a wide variety of mercantile-oriented reckoning contents and operations. As tailored to their audience and to the prevalent use of vernacular mathematics, the majority of reckoning examples show seemingly ordinary trade transactions, dealing with different goods or money exchange rates, and then opening up a math problem related to this mercantile activity. For instance, these examples, taken from reckoning books of the 15th and early 16th centuries, tell us about the trade with metal and spice:

[Note: Exposition] Item one buys 371 hundredweights of tin in Eger one hundredweight each for 10 guilders $\frac{3}{4}$ and the transportation charges and the customs of the journey to Nuremberg cost 121 guilders and one hundredweight in Nuremberg is sold for 8 guilders $\frac{1}{2}$ [Note: Instruction] Now you want to know what profit he has made with the tin alone you should know that one hundredweight in Eger weighs $133 \frac{1}{3}$ pounds in Nuremberg Now proceed to look at what 371 hundredweights cost and the result is 3 988 guilders $\frac{1}{4}$ Now add 121 [...] ⁹

[Note: Exposition] Note in Venice I have bought 4563 pounds of Ginger which have cost 24 ducats per 100 pounds including transportation charges and customs and what that has cost to go to Nuremberg Now he sells the ginger in Nuremberg one pound for 13 gros each [Note: Groschen] [Note: Instruction] Now I want to know how much I have profited of the 4563 pounds and consider that 60 pounds in Nuremberg are weighing 100 pounds in Venice and 100 ducats are worth 130 rhenish guilders now look at how much profit the 4563 generate Proceed as follows 100 pounds cost 24 ducats how much do 4563 pounds cost they cost 1095 ducats $4/25$ and thus convert this to guilders und see that 100 ducats convert to 130 rhenish guilders [...]¹⁰

The individual reckoning example can be divided in two parts, as highlighted in the examples: an exposition (Wiesinger 2023, 380) and an instruction, often consisting of a question, an approach and a solution (Gärtner 2002, 196; Feistner 2016a, 65–66). While the steps of the mathematical operation are taught in the second part of the example, the instruction, the preceding exposition serves as a kind of ‘stage’ conveying the variables and information necessary for solving the calculation. Examples can thus be divided into an assertive part, making a statement, and a directive part, prompting the recipients to perform an action (Searle 1976).¹¹ This transition is initiated by recurring phrases that shift the perspective from telling to directly addressing the recipient, indicating the transmission of an application-orientated “knowing how” (Krämer 2007, 43–45).

All mathematical examples found in the *practica* of a reckoning book are regulating a relevant practical context, namely mathematical knowledge useful for mercantile or trade-related professions. To do so, the examples make use of fictional case studies that generally appear to display realistic trade transactions or applicable scenarios involving wages, goods, or cooperations between merchants.¹² But they don’t necessarily have to tell an ‘economic’ story: This is demonstrated by the wide array of different possible expositions. To describe this, Kurt Vogel (1954, 155–156) uses the distinction between practice-oriented and recreational examples, the latter consisting of expositions designed to entertain and divert rather than simulate a close relation to everyday practice.¹³ While most reckoning examples employ seemingly practical expositions centered around mercantile activity, the exposition itself is therefore versatile and allows for nearly any kind of story. Consequently, some contents are not directly related to trade – but every example tells a story aiming to engage its recipients. To complete the representative corpus for this study, two additional examples were selected: one dealing with the siege of a castle, and another one showing an established recreational problem, often coined “The gatekeepers of the apple orchard” (Vogel 1954, 173–174). The first is taken from a reckoning book dated to 1547, while the second originates from a fifteenth-century reckoning book:

[Note: Exposition] Item 54 men were besieged in a castle the food supplies are enough for six months if everyone gets a daily ration of bread that weighs 24 lot after three months and twelve days they are alerted that they must make do with the remaining bread for another four months and thirteen days [Note: Instruction] How many lots of bread do they have to take away from every person’s daily ration to last the necessary time the month has 30 days for this calculation Now consider how much grain there is in the castle and what share is due to each individual [...]¹⁴

[Note: Exposition] Imagine a beautiful maiden had asked you to fetch her apples out of a walled garden that has many gates and the gatekeeper said that the garden is protected in a way that one must give him half of the collected apples and afterward also give his servant one additional apple from his collection of apples that he intends to bring to the maiden or else a pregnant lady and at every gate he must do the same so that in the end it only remains what he had promised to bring no more and no less [Note: Instruction] So first you have to consider how many gatekeepers there are and how much you must give to each servant and how many apples you are supposed to bring out If there are three gatekeepers and to each servant you give one apple and you are to bring out seven then take and double the 7 to make 14 and add twice as much as you must give to the first servant making 16 Now double the 16 to make 32 [...]¹⁵

Now that we have seen how versatile the contents of these short, practice-oriented expositions are, we might ask ourselves how we can best describe their narrative quality. To address this, the concept of liminal narratology as proposed by Matías Martínez (2023) can offer guidance: Following Martínez, even the shortest texts on the “threshold of narrativity” can be “rudimentary narrative” (Martínez 2023, 155) and therefore relevant for narratological analysis. To analyze them, one has to work with a more open conception of narration, considering a catalogue of optional and necessary elements of narrating a story (Martínez 2023, 157). In his article, Martínez assesses his sample text according to a wide range of criteria taken from essential narrative theories. While not all of these can be addressed in this study, a brief discussion will shed light on a few fundamental aspects in order to get a better grasp of the narrative character of the exposition.

The most fundamental requirement for narrativity is of course that the content shall be arranged in a temporal and causal sequence (Martínez 2017, 2–3). If we look at our exemplary problems, a very basic sequence is causally, temporally and spatially connected: Someone, presumably a merchant, buys goods and then transports them to another place to sell them again, where he has to factor in exchange rates in his trade. 54 men are under siege, have to ration their food, and as the conditions change, they are confronted with the challenge to make their remaining food last even longer. A man goes into an apple garden to retrieve a certain amount of apples and is then confronted with the challenge of meeting three guards and servants that each take away a portion. Most reckoning examples thus refer to a specific spatial and temporal context, which distinguishes them from many other pragmatic texts, such as medieval recipe treatises or artisanal guidebooks. The instructions found for instance in cookbooks or treatises on perspective and geometry do not convey factual knowledge to recipients through fictional case examples,¹⁶ but rather in a different form: through actionable guidelines for practice that are not tied to a specific place or a specific point in time, similar to the instructional part of the reckoning example, just telling the reader to do as follows.

The arrangement in a temporal and causal sequence is of course the most minimal and basic definition of narration.¹⁷ To look at further possible requirements, Martínez turns, among other things, to the axiological transgression in the sense of Lotman (Martínez 2017, 4–5): The overall narrative structure of a text is defined by a border that must be crossed for a narration to unfold. As

Renner (2004) shows, this border can take on various forms in the sense of being a topographical or political, but also a moral or normative boundary. Trade-related examples cross borders in several ways: Not only are geographical borders crossed when the merchant travels from Nuremberg, a center of European trade, to Eger [modern-day Cheb], a major hub for metal trade located on the route to Prague (cf. Hrubý 2024). These regions are also divided by different coin systems, political borders and their own complex supply and demand system – thus geographical as well as political and economic borders are crossed. The recreational example, on the other hand, tells of the crossing of a spatial boundary between an undefined exterior and a walled-in, carefully guarded interior, the apple orchard; the example of the siege establishes a spatial boundary between the besiegers and the besieged, but the more important boundary is the temporal one, which is linked to the rationing of the remaining grain and therefore the question of survival or defeat.

At the level of textual composition, certain markers signaling narrativity are thus indeed present. However, the expository narration of the reckoning example lacks a crucial feature: closure. It does not unfold as a complete sequence with a clear beginning, middle, and end. Instead, the storyline is developed only up to the point at which the mathematical procedure can be introduced, after which it is abruptly abandoned in favor of the instruction.¹⁸ Correspondingly, the beginning is also abrupt: The story begins right at the point where the variables for the equation can be conveyed, without any more introduction or build-up than necessary. Only the mathematical operation itself – introduced and prepared by the exposition – reaches a satisfactory conclusion, while the story remains incomplete. Many gaps are never filled: How does the trader use his profit and what does he purchase with his money? Did the man get the amount of apples, and why did the maid require them in the first place? Was the siege of the castle successful and who were the warring parties? Such possible continuations, as well as embellished beginnings or additional details, are left unexplored. The narration is thus deliberately streamlined, serving to establish only the necessary setting in which the calculation can take place. The story that is written down is purposely reduced to the minimal narrative framework necessary for the calculation to unfold. Expositions on the brink of narrativity can therefore, following Martínez' established term, be described as “narratogenic” (Martínez 2023, 155). Yet, despite their brevity in form and content, this does not hinder the effectiveness of the narration as a didactic tool. On the contrary, it is exactly their concision that makes the examples so productive: by stimulating the imagination of their readers.

3. Less Is More – Evoking Protonarratives

Reckoning examples are, as many narrative elements in pragmatic literature, intended to engage readers, capture their interest and thereby enhance the didactic

effect of the text. How do they manage to meet these requirements even within their limited, narratogenic form? This is achieved because the short form of the example is cleverly used to draw on the audience's existing knowledge of the world and of contemporary discourses. Their brief stories are sufficient not only because they aptly present the variables of a reckoning operation, but also because the examples are artfully crafted to require only a few words to activate pre-existing cultural and worldly knowledge, thus becoming immediately meaningful in the reader's mind.

Analyzing our examples, we can identify knowledge possibly embedded in the exposition: The manuscript source of the trade example from Eger to Nuremberg appears to have been written by a single hand and includes a wide variety of German-language examples which are mostly directly related to trade. A few Italian examples added on the last pages additionally allude to contact with Italian merchants and mathematics and show the necessity of knowledge related to long-distance trade. This is further underlined by the frequent mentioning of Venice in several examples, as the city was the dominant force on the Mediterranean trade market and the central hub of European trade (cf. Pfothenhauer 2016). It also repeatedly mentions other important "trading hubs" (Denzel / Häberlein 2024, 191), namely Nuremberg, Eger, Cologne, Pressburg and Innsbruck, all located in or directly connected to the Southern German trade region. Most reckoning examples focus on long-distance trade by calculating money exchange rates and measurement differences between these cities, thus portraying not only a network of cities, but also the most popular commercial goods mentioned by Denzel and Häberlein (2024): Metals and cloth from Central Europe, luxury goods such as saffron, pepper and ginger arriving from the Mediterranean.¹⁹

The example tells us about the trade of tin from Eger to Nürnberg: For the contemporary reader, the mere mention of these places and goods is likely enough to evoke pre-existing worldly knowledge. Nuremberg was of course one of the most central trading hubs in Europe, and Eger in Bohemia was a vital place for the trade with tin, because ore mining was of significant importance in the surrounding regions (cf. Hrubý 2024).²⁰ It is furthermore located directly on the potent trade route from Nuremberg to Prague (cf. Kellenbenz 1971). Trade between the two cities and trade privileges were thus already established in the 14th century, and after periods of disruption, it again reached a new prosperity in the second half of the 15th century (cf. Kaar 2021). The other example, relating to the trade of ginger from Venice to Nuremberg, touches on the most important exports from the 'Serenissima': the trade of spices and 'Spezereien', various exquisite seasonings and delicacies. Venice was the most important trading center for Nuremberg merchants, where goods from the Levant, Central Europe, and Northern Europe converged (Pfothenhauer 2016, 265–270). Of all the spices, ginger was, along with pepper, saffron and cloves, the most sought-after spice and the most important commodity in trade between Nuremberg and Venice (Pfothenhauer 2016, 270). The ginger trade from Venice is therefore a staple of the Nuremberg merchant's business.

These problems convey distinctly mercantile knowledge in several ways: First and foremost, they draw on the practical knowledge of trade connections between cities, regional supply and demand, and the conditions under which profitable exchanges emerge. For instance, where tin is to be bought and how it can be successfully transported and sold off again – similar knowledge of trade and trading hubs is both recalled and reinforced in many examples of reckoning books. Therefore, there is no need to spell out much more explicitly in the individual examples. At the same time, they present a larger narrative of the world ruled and structured by commerce and mercantile activity. Since stories and narratives extend across all spheres of societal significance (Koschorke 2013, 18–19), they also shape societal models:

In politics as well as in jurisprudence, the significance of minor and major narratives is unmistakable – ranging from case studies [...] all the way to founding myths [...]. And ultimately, even economic models can to a large extent be defined as arrangements that rest on the acceptance of certain narratives [transl. by the author] (Koschorke 2013, 19–20).

Reckoning books display a prevalent societal narrative of the early modern time through their examples: The model of a world dominated by a thriving economy, and thus by mathematical knowledge. Correspondingly, economic topics are not only addressed in the wide variety of reckoning books, but also grow increasingly important in the early modern vernacular literature: For instance, the first German-language prose novel, the *Fortunatus* (1509), tells the story of a man making his own fortune in a modern, globalized world dominated by merchantry.²¹

Conversely, the economic sphere is permeated and sustained by narratives. Reckoning examples both draw on and reinforce the presumed knowledge of their recipients: the knowledge of trade hubs and trade routes, of monetary systems and of the interplay between political and economic borders. In doing so, they also activate bigger narratives about a functioning and prospering market economy, the power of long-distance trade surpassing feudal and political borders and a world governed by the dynamics of supply and demand, already exhibiting ‘protocapitalist’ structures (cf. DuPlessis 2019). The depicted significance of number-related knowledge is furthermore not restricted to the sphere of trade, but is connected to the structure of the world itself in the prefaces of many reckoning books. Citing the biblical sentiment that God created all things “according to number, weight and measure”,²² the art of arithmetic is then depicted as a fundamental knowledge, essential for navigating and comprehending all domains of worldly activity.²³ Arithmetical knowledge is therefore elevated above anything else while simultaneously valorizing the economic domain.

Reckoning examples are thus able to articulate a larger narrative in which merchantry, finance and mathematics permeate the aspects of life, suggesting that reality itself can be structured and must be understood through arithmetic knowledge. Expositions introduce these narratives to the readers and relate this knowledge in a manageable, playful, rehearsing manner. The way these narratives are transported even in the very concise expositions of reckoning examples is aptly explained with the concept of “protonarratives”, established by Jan-Dirk

Müller (2007, 23–28). Müller (2007, 24) defines protonarratives as incomplete segments or set pieces of narrations: They are “subsegments of narratable sequences or problem or conflict constellations that can be unfolded narratively [transl. by the author]”. As such, they can be ‘planted’ into a text and then unfold in the minds of their readers, as their mentioning alone is able to connect to a larger narrative without having to spell out the ‘whole story’ they allude to. Protonarratives originate in the understanding and the knowledge of the world, and in the discourses that are known to their intended audience (Heilig 2020, 548–556). Reckoning examples play exactly at that knowledge to work even in their succinct form: For instance, if Eger is mentioned in an example as a place where a merchant buys tin ore, the knowledge of Bohemia as a metal supplier, of Eger as a potent trading hub specialized in tin, and the location of the city on a frequented trade route from Nuremberg to Prague is called upon in the mind of the reader, and larger narratives of merchant practices and trade connections are set in motion. By doing so, they also outline spaces of experience and possibility, fashioning a self-image of the readers positioned as an active part of the larger mercantile narrative.²⁴

Protonarratives can refer to mercantile knowledge and thus paint the picture of a world driven by trade and economy, but since they are variable, they can take on any kind of recognizable narrative: Thus one can also ask to what extent arithmetic examples provide space for the transmission of other knowledge, e.g., social and cultural discourses (cf. Orbán / Wiesinger 2024). To examine the third example, the men under siege, in this light, a closer look at the source is again worthwhile: The scribe is a certain “Enderes Beham”, Andreas Behaim of Nürnberg.²⁵ Behaim was a scholar of the famous reckoning master Johann Neudörffer the Elder (*1497; †1563) and authored his reckoning book starting from the year 1547,²⁶ seemingly for collecting and preserving valuable arithmetic knowledge. Amongst an abundance of goods, exchange rates and trade examples, the exposition of the siege is located on the last written page of the main body of text. Afterwards, there are two addenda: One chronicling Behaim’s reckoning teachers and travels, and one chronicling the siege of Nuremberg during the Second Margrave War (1552–1555) in 1552.²⁷ During the war, Nuremberg and other free cities fought with the Margrave for dominance over Franconia. The addendum describes how the city was unsuccessfully besieged and how the army of the Margrave laid waste to the surrounding landholdings of the city. This was only a culmination of long-standing sweltering conflicts between the free imperial city of Nuremberg and the Margrave as well as the surrounding knightly nobility (cf. Hirschmann 1971 / Schnellbögl 1971). This „ever-present threat to urban life posed by external violence, constantly felt by the city’s inhabitants [transl. by the author]“ (Meyer 2009, 351) in the 15th and 16th centuries was likely also the context against which such an example should be read – not as a direct guide on how to behave, should the emergency arrive, but as an allusion to the contemporary circumstances surrounding the city, present in the minds of the scholars and students, and the potential necessity of calculating rations. Omitting certain expositions or adding them to the *practica* of a

reckoning book is always a decision in favor of or against a particular narrative²⁸ – in this case, a narrative that does not omit the possibility and looming presence of conflict, or, as Hans Sachs states, of the city of Nuremberg as a *hortus conclusus*, a paradise permanently surrounded by enemies (Keller 1870, 189–199).²⁹

Recreational problems such as the apple orchard example can also incorporate various protonarratives: First of all, on a normative factual level, they convey a useful mathematical operation much the same as a practical example would. In the case of the orchard example, the repeated payment of an equal sum is reminiscent of annually due payments, or several custom or interest payments (Tropfke 1980, 582). The mathematical operation itself, enclosed in various expositions of examples (e.g. sharing a pile of coconuts and always giving a tribute to a monkey; or laying down parts of a flower arrangement at several temple gates),³⁰ already appeared in mathematical texts since antiquity and has continued to be a popular and widespread operation until the late Middle Ages (Tropfke 1980, 582–583 / Folkerts 1971, 69–71). Thus, at its core it transmits a popular practical application. The story told in the exposition itself, however, can connect to a socio-cultural instead of an economically driven narrative: In German-vernacular reckoning books, the operation is remarkably often wrapped in the apple-orchard narration. This is because the combination of the walled garden, and the suggestion of romantic or erotic love is able to evoke several cultural discourses likely recognizable and therefore appealing to readers. For one of the similar examples appearing in another vernacular reckoning book, Edith Feistner observes that the orchard examples uses “topoi of a love story” (Feistner 2016b, 89).

I want to take this a step further and briefly highlight the connection to cultural narratives that are implanted in this version of the example. The orchard, the apple and the walled garden can be linked to several by then long-standing topoi: On one hand, the garden is well-established as a site of pleasure and of romantic, potentially forbidden encounters in medieval and early modern literature (cf. Frühe 2002 / Krohn 2008). In particular, the pleasure garden, often described as an orchard surrounded by walls, recurs in medieval literature as a site associated with the topos of the *locus amoenus* and a setting of (illicit) love (Lichtblau 2008, 502–509). The connection between a walled orchard, an apple as a token of love and the maid demanding this apple can also be understood as an allusion to the biblical story of Adam, Eve and the First Sin – possibly a humorous innuendo to the erotic, perhaps sinful relationship between the man and the maiden. The apple as a token of lovers is separately likewise established in early modern discourse: It repeatedly appears as an erotic attribute in various forms, for example in early modern emblematic works. In the 1542 version of Andrea Alciato’s popular emblem book,³¹ the ideal marriage is depicted in text and image through a couple with two attributes: Marriage is shown as a union of fidelity, symbolized by the dog at the woman’s feet, and passion, embodied by the apple in her left hand. In the accompanying Latin and German epigrams, the apple is not only called a “heated fruit” but also a “fruit of Venus”, credited with inciting erotic passion.³² All of this knowledge is not necessarily needed to

understand, replicate and practice the mathematical operation in the example. But it serves to make a humorous and diverting story out of only a minimal narration, and thus makes the example more memorable and didactically useful for those who understand the bigger story hidden in it.

Again, as in the other examples, it activates protonarrative structures that evoke associations already present in the text, without articulating them explicitly. As Müller argues, protonarratives are preliminary narrative forms that serve as the very condition for literary narration: Pre-narrative knowledge exists on a level prior to textual articulation, yet it nonetheless serves as a foundation of storytelling (Müller 2007, 24). Arithmetic examples such as the apple orchard example draw precisely on these protonarratives and take advantage of the limited exposition space by evoking associations that deliberately play with the audience's experiences and expectations. From only a few words, an entire, bigger story is set into motion – a story not written on the page, but generated by the reader through their own associative, imaginative connections.

The expositions can therefore be reduced to the most minimal, narratogenic form and yet conjure so much: They activate existing knowledge and associations while at times also fostering new connections, whether it be an allusion to economic knowledge or to socio-cultural discourses. Reckoning examples impart and reinforce mathematical as well as worldly knowledge. It is precisely their stories, created by the recipients as they fill in the gaps, that engage the readers and thus simultaneously reinforce the didactic effect. What we encounter in the arithmetic books is therefore the shell of a potentially full-blown story that does not work without the calculation but necessarily complements it: Expositions succeed in conveying only what is necessary to make the narrative comprehensible while also being appealing and engaging for the recipients. By utilizing the allusive power of protonarratives, expositions also manage to make use of one of the positive effects of a good story: Transmitting knowledge through a story is a powerful mnemonic and didactic tool, and often simply more memorable than just pure instruction (Carruthers 2008, 184, 223–225).

4. The Puzzle of Mathematics

According to Müller (2007, 25), protonarratives can furthermore also serve as the basis for the formation of simple forms (cf. Jolles 2006). In this respect, the simple form of the riddle presents a particularly interesting parallel to the reckoning example, as they are both designed to convey knowledge.

Jolles defines the riddle at its core as a question wanting to be answered and links this to the desire to instill knowledge (Jolles 2006, 129–130). Reckoning examples are therefore, in line with their discursive function as an individual example for a general mathematical operation, akin to riddles as they also serve the transmission of knowledge in the form of a task designed to be solved. This mode of presenting knowledge is the common denominator between a riddle

and a reckoning example: The recipient wants to solve the question the riddle or example poses and the inquirer wants to pass on his knowledge through this assignment (Wiesinger 2023, 381 / Jolles 2006, 129–130).³³ It is therefore no coincidence that Jolles (2006, 131–132), focussing on the factor of knowledge transfer, also draws a parallel between the riddle and the exam, where again a question is posed to be solved, both to test and to reinforce knowledge in a didactic setting. All formats, the riddle, the exam question and the reckoning example, also share an inherently dialogic nature: The ‘interrogator’ or enquirer shares a task that can and should be answered by the responder (Wiesinger 2023, 381). In this case, the position of the enquirer is taken on by the mediated voice of the reckoning master that serves as the narrator of the examples, while the respondent is the recipient of the reckoning book, may it be a scholar or a merchant using the book to retain and practice mathematics.

There is, however, a significant difference between an exam question and a riddle or reckoning example which does not lie in the fact that knowledge is tested, but rather in the manner this test is presented. In the exam question, the scholar is tasked with answering a direct, straightforward question, relying on his expertise. In the riddle or the reckoning example on the other hand, this is not the case: The information required to solve the task is encrypted in a question or, in the case of the reckoning example, in the exposition. Consequently, a process of decipherment must take place (cf. Tomasek 1994): One needs to decode the metaphorical language of the riddle to extract the information necessary to solve it. Similarly, the student engaging with the reckoning example has to determine which elements of the story in the exposition can be translated into variables and how these variables may be integrated in a reckoning operation capable of producing the desired result.

If we look at the apple orchard example, its exposition includes several variables that have to be extracted from the story and put into the right order for the equation to work: A variable for the amount of apples taken away by each guard at each gate, a variable for the additional amount of apples taken away by the servant, the number of gates one has to go through and a variable for the initial quantity of apples the man left the garden with in the first place. These components have to be extracted out of the story and arranged correctly in order to construct an equation that solves the initial question: How many apples does one need to pluck in the first place? All these variables are already present in the examples: not, as in a riddle, encoded through ambiguous terms, allusions or metaphors, but through the transformation of mathematical variables into narrative building blocks. These must be ‘decoded’ by the students themselves, so to say, be identified and correctly applied in a mathematical operation. The dialogicity that instigates this process fittingly comes into play at the transition from exposition to instruction: After the story is over, there is a shift in mediacy, as the voice ceases to narrate, shifts its focus and talks directly to the recipient (Feistner 2016a, 65–66) hinting at what to look for: “So first you have to consider how many gatekeepers there are and how much you must give to each servant and how many apples you are supposed to bring out [...]”

This process of encoding or decoding is not only the most remarkable characteristic of the riddle (cf. Tomasek 1994), but similarly also of reckoning examples: Arithmetic examples are evocative of solvable riddles, combining the processes of encryption and decryption with an element of dialogicity explicitly reflected in the simulated orality of a reckoning master instructing a student. The exposition is the place that sets up the riddle by encoding all its valuable parts in the form of a short, but effective story. Understanding the study and then independently coming to a conclusion on one's own is a skill that taught and transmitted through the narrative.

Against the backdrop of this similarity, it comes as no surprise that in early medieval times, arithmetic knowledge was frequently conveyed in the form of entertaining riddles (cf. Folkerts 1978). As “mental dispositions” (Jolles 2006, 128) that “configure and ‘solve’ everyday problems in a specific way” (Müller 2007, 25), both the riddle and the reckoning example demonstrate similar abilities: In the case of the reckoning example, necessary and important mathematical knowledge is structured into manageable units and conveyed, stored, and passed on in such a way that it remains applicable and didactically effective.

5. Conclusion

The reckoning examples examined in this article have shown that a close, intertwined relationship between narrative and non-narrative parts is a foundational element in the vernacular transmission of mathematical knowledge. In the examples, the expositions provide the information necessary to perform the calculation, primarily in the form of a concise, narratogenic story. Now one might ask why these examples do not simply present the relevant variables in a merely descriptive or even listed form: Why tell of merchants transporting tin from Eger to Nuremberg, of people besieged in a castle, or of a lover retrieving apples from an orchard, when the calculation itself could be initiated more economically?

The answer lies in the complex functionality that the narrative part of the example fulfills for its readers. Presenting knowledge in a story is first of all simply more delightful and engaging: And even in their narratogenic, highly condensed and often incomplete form, the expositions are able to stimulate larger narrative structures and to activate what Müller (2007, 23–25) describes as “protonarratives”. Their quality invites readers to fill in the gaps themselves, by drawing on a shared knowledge of the world and connecting the example to broader economic, political, or cultural contexts. Whether the exposition evokes references to daily practices or to well-known literary and cultural tropes, it sparks interest, captures attention and, in doing so, enhances precisely the mnemonic processes on which successful knowledge transfer depends.

The examples therefore demonstrate that didactic communication is rarely based on instruction alone: Reckoning books use narration to keep their recipients focused and immersed to convey knowledge more effectively. These

important characteristic parallels the reckoning example with the riddle according to Jolles (2006) and Tomasek (1994), as both the riddle and the reckoning example encode knowledge in a way that it can and must be deciphered, thus transforming learning into an active, engaging process.

In conclusion, the narrative forms encountered in reckoning books occupy a liminal position at the threshold of storytelling: They are proto-narrative structures that remain deliberately incomplete, yet they are highly effective exactly because of this brevity. The calculation cannot function without the framework of a story, while the narration would remain an empty shell without the calculation that gives it purpose. And even in this concise form, the examples not only succeed in presenting mathematical knowledge in an engaging and practical way, but they also tell a larger narrative about the value and significance of mathematics as a whole.

Bibliography

- Bauschke-Hartung, Ricarda et al. (eds.) (2017): *Die Kunst der ‚brevitas‘. Kleine literarische Formen des deutschsprachigen Mittelalters*. Rostocker Kolloquium 2014. Berlin.
- Burnett, Charles (2010): *Numerals and Arithmetic in the Middle Ages*. New York, NY.
- Caflich, Sophie (2018): *Spielend lernen. Spiel und Spielen in der mittelalterlichen Bildung*. Ostfildern.
- Carruthers, Mary (2008): *The Book of Memory. A Study of Memory in Medieval Culture*. Cambridge.
- CoReMA. *Cooking Recipes in the Middle Ages*. URL: <https://gams.uni-graz.at/context/corema> (29.04.2026).
- Denzel, Markus / Häberlein, Mark (2024): *Handbuch globale Handelsräume und Handelsrouten. Von der Antike bis zur Gegenwart*. Berlin et al.
- Doering, Pia Claudia / Emmelius, Caroline (eds.) (2017): *Rechtsnovellen. Rhetorik, narrative Strukturen und kulturelle Semantiken des Rechts in Kurz Erzählungen des späten Mittelalters und der Frühen Neuzeit*. Berlin.
- DuPlessis, Robert (2019): *Transitions to Capitalism in Early Modern Europe. Economies in the Era of Early Globalization, c. 1450–c.1820*. Cambridge.
- Ewert, Ulf Christian et al. (2023): “Räumliche Konfigurationen des Messehandels im frühneuzeitlichen Europa.” In: *Vierteljahrschrift für Sozial- und Wirtschaftsgeschichte* 110 (No. 4), pp. 494–541.
- Faschingeder, Kristian / Reisenbichler, Anna (2025): “‘Dürer für Dummies’. Eyn schön nützlich büchlin und underweisung der kunst des Messens.” In: Michaela Wiesinger (ed.), *Knowledge and Order. Monastic Learning and the Quadrivium*. Vienna, pp. 181–201.
- Feistner, Edith (2016a): “Geschichten zum Rechnen – Geschichte des Rechnens (1): mathematische Textaufgaben in narratologischer Perspektive.” In: Edith Feistner / Alfred Holl (eds.), *Erzählen und Rechnen in der frühen Neuzeit. Interdisziplinäre Blicke auf Regensburger Rechenbücher*. Berlin, pp. 63–78.
- Feistner, Edith (2016b): “Geschichten zum Rechnen – Geschichte des Rechnens (2). Konturen der Narration in mathematischen Textaufgaben der frühen Neuzeit.” In: Edith Feistner / Alfred Holl (eds.), *Erzählen und Rechnen in der frühen Neuzeit. Interdisziplinäre Blicke auf Regensburger Rechenbücher*. Berlin, pp. 79–118.
- Folkerts, Menso (1971): “Mathematische Aufgabensammlungen aus dem ausgehenden Mittelalter. Ein Beitrag zur Klostermathematik des 14. und 15. Jahrhunderts.” In: *Sudhoffs Archiv* 55 (No. 1), pp. 58–75.
- Folkerts, Menso (1978): *Die älteste mathematische Aufgabensammlung in lateinischer Sprache. Die Alkuin zugeschriebenen PROPOSITIONES AD ACUENDOS IUVENES*. Überlieferung, Inhalt, Kritische Edition. Vienna.
- Folkerts, Menso / Hughes, Barnabas (2016): “The Latin Mathematics of Medieval Europe.” In: J. Lennart Berggren et al. (eds.), *Sourcebook in the Mathematics of Medieval Europe and North Africa*. Princeton, NJ, pp. 4–223.

- Frühe, Ursula (2002): *Das Paradies ein Garten - der Garten ein Paradies. Studien zur Literatur des Mittelalters unter Berücksichtigung der bildenden Kunst und Architektur*. Frankfurt a.M. / Vienna.
- Gärtner, Barbara (2000): *Jobannes Widmanns 'Bebende vnd hupsche Rechenung'. Die Textsorte 'Rechenbuch' in der Frühen Neuzeit*. Tübingen.
- Haustein, Heinz-Dieter (2004): *Quellen der Meßkunst. Zu Maß und Zahl, Geld und Gewicht*. Berlin.
- Heilig, Christoph (2020): *Paulus als Erzähler? Eine narratologische Perspektive auf die Paulusbriefe*. Berlin / Boston.
- Herrmann, Dietmar (2016): *Mathematik im Mittelalter. Die Geschichte der Mathematik des Abendlands mit ihren Quellen in China, Indien und im Islam*. Berlin / Heidelberg.
- Herrmann, Dietmar (2024): *Die antike Mathematik. Geschichte der Mathematik in Alt-Griechenland und im Hellenismus*. Berlin / Heidelberg.
- Hirschmann, Gerhard (1971): "Zeitalter des Markgrafen Albrecht Achilles." In: Gerhard Pfeiffer (ed.), *Nürnberg – Geschichte einer europäischen Stadt*. Munich, pp. 115–120.
- Hrubý, Petr (2021): *Erzbergbau und Edelmetallproduktion im böhmischen Königreich während des 13. Jhs. im Kontext der europäischen Montanarchäologie*. Dresden.
- Hrubý, Petr (2024): *Silver Mining in the Kingdom of Bohemia (13th–14th Centuries)*. Boston / Leiden.
- Jackel, Christina et al. (2023): "Handwritten Arithmetic Treatises in German (1400–1550). A First Assessment of the Sources Based on the Exemplary Corpus Held by the Austrian National Library." In: *Medieval Worlds* 18, pp. 72–107.
- Jolles, André (2006): *Einfache Formen. Legende, Sage, Mythe, Rätsel, Spruch, Kasus, Memorabile, Märchen, Witz*. 8th ed. Tübingen.
- Kaar, Alexandra (2021): "The 'Golden City' Under Embargo: Prague's International Trade During the Hussite Wars, 1420–1436." In: *Central European History* 54 (No. 3), pp. 447–465.
- Kellenbenz, Hermann (1971): "Gewerbe und Handel am Ausgang des Mittelalters." In: Gerhard Pfeiffer (ed.), *Nürnberg – Geschichte einer europäischen Stadt*. Munich, pp. 176–186.
- Keller, Adelbert von (ed.) (1870): *Hans Sachs. Vierter Band*. Tübingen.
- Klein, Christian / Martínez, Matías (eds.) (2009): *Wirklichkeitserzählungen. Felder, Formen und Funktionen nicht-literarischen Erzählens*. Stuttgart.
- Koschorke, Albrecht (2013): *Wahrheit und Erfindung. Grundzüge einer Allgemeinen Erzähltheorie*. Frankfurt a.M.
- Kössinger, Norbert / Wittig, Claudia (eds.) (2019): *Prodesse und delectare. Case Studies on Didactic Literature in the European Middle Ages / Fallstudien zur didaktischen Literatur des europäischen Mittelalters*. Berlin / Boston.
- Krämer, Sybille (2007): "Mathematik zwischen *knowing how* und *knowing that*. Überlegungen zu zwei Innovationsschwellen des mathematischen Denkens." In: Klaus W. Hempfer / Anita Traninger (eds.), *Dynamiken des Wissens*. Freiburg i. Br. et al., pp. 43–60.
- Krohn, Rüdiger (2008): "Ein boumgart umb das hûs lac. Höfische Natur als Schonraum der Liebe in der deutschen Literatur des Mittelalters." In: Ulrich Müller (ed.), *Burgen, Länder, Orte*. Konstanz, pp. 91–108.
- Laube, Stefan (ed.) (2024): *Tracts of Action. Material, Visual, and Practical Dimensions of Early Modern How-to Books*. Leiden / Boston.
- Leidinger, Hannes (2008): *Kapitalismus*. Stuttgart.
- Lichtblau, Karin (2008): "Locus amoenus. Der 'liebliche Ort' – ein Topos in der Literatur des Mittelalters." In: Ulrich Müller (ed.), *Burgen, Länder, Orte*. Konstanz, pp. 497–510.
- Martínez, Matías (2017): "Was ist Erzählen?" In: Matías Martínez (ed.), *Erzählen. Ein interdisziplinäres Handbuch*. Stuttgart, pp. 2–6.
- Martínez, Matías (2023): "Nullstufen und Vorstufen des Erzählens. Überlegungen zu einer liminalen Narratologie." In: *Anglistik. International Journal of English Studies* 34 (No. 2), pp. 155–163.
- Meyer, Carla (2009): *Die Stadt als Thema. Nürnbergs Entdeckung in Texten um 1500*. Ostfildern.
- Müller, Jan-Dirk (2007): *Höfische Kompromisse. Acht Kapitel zur höfischen Epik*. Tübingen.
- Orbán, Norbert Hunor (2025): "Faith in Trade. The Reckoning book of Clemens Hör (1515–1572) and the Swiss Reformation Crisis." In: Michaela Wiesinger (ed.), *Knowledge and order. Monastic Learning and the Quadrivium*. Vienna, pp. 45–62.
- Orbán, Norbert Hunor / Wiesinger, Michaela (2024): "Zwischen den Zeilen gerechnet. Rechenbeispiele als Spiegel sozialhistorischer Entwicklungen im 15. und 16. Jahrhundert." In: *Memo. Medieval and Early Modern Material Culture Online* 11. URL: <https://memo.imareal.sbg.ac.at/wsarticle/memo/zwischen-den-zeilen-gerechnet/> (15.03.2026).
- Pfotenhauer, Bettina (2016): *Nürnberg und Venedig im Austausch. Menschen, Güter und Wissen an der Wende vom Mittelalter zur Neuzeit*. Regensburg.

- Renner, Karl Nikolaus (2004): "Grenze und Ereignis. Weiterführende Überlegungen zum Ereigniskonzept von Jurij M. Lotman." In: Gustav Frank / Wolfgang Lukas (eds.), *Norm – Grenze – Abweichung. Kultursemiotische Studien zu Literatur, Medien und Wirtschaft*. Passau, pp. 357–381.
- Rothmann, Michael (2001): "Überall ist Jahrmarkt. Entwicklungstendenzen der Institution des periodischen Marktes in Zentraleuropa vom 14. bis zum 17. Jahrhundert." In: Simonetta Cavaciocchi (ed.), *Fiere e mercati nella integrazione delle economie europee, secc. XIII–XVIII. Atti della Trentaduesima Settimana di Studi' 8–12 maggio 2000*. Florence, pp. 91–108.
- Schnelbögl, Fritz (1971): "Zwischen Zollern und Wittelsbacher." In: Gerhard Pfeiffer (ed.), *Nürnberg – Geschichte einer europäischen Stadt*. Munich, pp. 120–127.
- Searle, John R. (1976): "A Classification of Illocutionary Acts." In: *Language in Society* 5 (No. 1), pp. 1–23.
- Singmaster, David (2004): "Sources in Recreational Mathematics. An Annotated Bibliography. Eighth Preliminary Edition." *Assorted Articles on Recreational Mathematics and the History of Mathematics* (19.03.2004). URL: https://www.puzzlemuseum.com/singma/singma6/SOURCES/singma-sources-edn8-2004-03-19.htm#_Toc6953410 (28.04.2026).
- Smolnik, Regina (ed.) (2014): *Silberausch und Berggeschrey. Archäologie des mittelalterlichen Bergbaus in Sachsen und Böhmen. Stříbrná borečka a volání hor. Archeologie středověkého hornictví v Sasku a Čechách*. Dresden.
- Tomasek, Tomas (1994): *Das deutsche Rätsel im Mittelalter*. Tübingen.
- Tropfke, Johannes (1980): *Geschichte der Elementarmathematik. Band 1: Arithmetik und Algebra. Vollständig neu bearbeitet von Kurt Vogel, Karin Reich, Helmuth Gericke*. 4th ed. Berlin / New York, NY.
- Vogel, Kurt (1954): *Die Practica des Algorismus Ratisbonensis. Ein Rechenbuch des Benediktinerklosters St. Emmeram aus der Mitte des 15. Jahrhunderts nach den Handschriften der Münchner Staatsbibliothek und der Stiftsbibliothek St. Florian*. Munich.
- Wiesinger, Michaela (2023): "Regula Virginum: Die kulturhistorische Relevanz von Rechenaufgaben am Beispiel des 'Zechrechnens.'" In: *Archiv für Kulturgeschichte* 105 (No. 2), pp. 377–404.
- Wußing, Hans (2008): *6000 Jahre Mathematik. Eine kulturgeschichtliche Zeitreise. I: Von den Anfängen bis Leibniz und Newton*. Berlin / Heidelberg.

Franziska Putz, MA

ERC Starting Grant ARITHMETIC

Institute of German Studies, University of Innsbruck

Innrain 52d, 6020 Innsbruck

E-mail: franziska.putz@uibk.ac.at

Project Website: <https://www.arithmetic-project.org/>

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¹ There is no way to provide a comprehensive overview of the German studies research on pragmatic writing and short text forms that has influenced this article; therefore, only a few highlights can be presented here: Kössinger / Wittig (2019); Laube (2024); Doering / Emmelius (2017); Bauschke-Hartung et al. (2017).

² This article is based on a presentation given at the 8th Conference of the European Network of Narratology on the topic of “Limits of Narrative.” I would like to extend my special thanks to the panel leaders Prof. Dr. Rabea Kohnen and PD Dr. Coralie Rippl-Uhlenhut, who gave me the opportunity to turn my presentation into an article. The study of late medieval and early modern reckoning books from a philological perspective is also the broader subject of my dissertation “Narrating knowledge. The late medieval and early modern reckoning book between narration, fiction and didactics”, which is a part of the ERC project “ARITHMETIC. German Arithmetical Treatises in Manuscripts of the Late Middle Ages (1400–1522). A Study on Philology, History and Culture based on a Digital Edition of the Treatises” (<https://www.arithmetic-project.org/>). The approaches outlined in this article are discussed in greater detail in the dissertation.

³ The large number of trade fairs and markets demonstrates the importance of mercantile activity in the 15th and especially the 16th century (cf. Rothmann 2001).

⁴ During his stays at a trading post in Algeria [modern-day Bejaia], Leonardo came into contact with the Hindu-Arabic numerals. These numbers had spread primarily through the translated

works of the Arab mathematician al-Khwārizmī, whose name also gave rise to the term ‘algorithm’, used for mathematical treatises with Hindu-Arabic numerals (Burnett 2010, 15–26).

⁵ Previously, Roman numerals had been used for reckoning operations, but Hindu-Arabic numerals replaced them simply because they were much better suited for performing complex arithmetic operations (Vogel 1954, 4). This is, among other things, due to the fact that they operate on the decimal system. Consequently, they allow for easier written calculations and calculating with larger amounts.

⁶ The project ARITHMETIC is, amongst other things, concerned with transcribing and describing German vernacular arithmetic manuscripts from the late Middle Ages, and is currently working on 116 manuscripts. Additionally, the project looks into various printed arithmetic books from the 15th and 16th centuries. The project’s fundamental questions concerning cataloguing, contextualization and transcription are described in an article by Christina Jackel, Norbert Hunor Orbán and Michaela Wiesinger (2023).

⁷ Many works on the history of mathematics deal with the origins and occurrences of arithmetic operations and examples: Herrmann (2016); Tropfke (1980); Wußing (2008).

⁸ Major works on reckoning books from a philological perspective include Barbara Gärtner’s (2000) text-linguistic analysis of Johannes Widmann’s arithmetic book (1489), and Edith Feistner’s narratological analyses of early modern arithmetic books (Feistner 2016a; Feistner 2016b).

⁹ Berlin State Library – Prussian Cultural Heritage Foundation, Ms. germ. oct. 375, 64r–64v. “Item einer kaufft 371 centner tzynns / Zu Eger Je 1 centner umb 10 gulden / 3/4 und kost Zu fur lon und / zu zol gen Nurmberg 121 gulden / und gibt 1 centner Zu Nurmberg / umb 8 gulden 1/2 Wiltu nu wissen / was er gewyns an dem tzin / allein hab du solt wissen / das 1 centner Zu Eger wigt Zu / Nurmberg 133 pfund 1/3 davon / so machs also wart wivil / 371 centner kosten unnd kumen 3 / 988 gulden 1/4 Nu addir 121 / [...]”

¹⁰ Munich, Bavarian State Library, Cgm 4142, 19r. “Merck ich han kauft zu venedig 4563 pfund / Ingwer die kumen ye 100 pfund umb 24 duckaten / mit furlon und mit zollen und waz den kost / hat uncz gen nerenberg Nü der verkauft die / Ingwer zu nerenberg ye ein pfund umb 13 gros / Nü wil ich wisen waz ich gewin an den 4563 pfund / und wisz daz 60 pfund von nerenberg wigt 100 pfund / zu venedig und 100 duckaten gelten 130 Rheinisch gulden / machs also wart wie vil die 4563 pringent / Sprich also 100 pfund chost 24 duckaten wie / kumen 4563 pfund die kumen 1095 ducaten 4/25 und / do mach den gulden dar ausz und machs also / 100 ducaten gelten 130 Rheinisch gulden [...]”

¹¹ Gärtner establishes a similar classification for mathematical text problems: According to Gärtner, individual problems consist of a communicative part – the actual problem statement, which corresponds to the concept of exposition – and an instructive part, guiding the reader to solve the calculation (Gärtner 2000, 186).

¹² The use of fictional case studies to support factual claims is reminiscent of Klein’s and Martínez’ (2009) concept of ‘Wirklichkeitserzählungen.’ The mathematical examples shall therefore also be considered as ‘Wirklichkeitserzählungen,’ since they illustrate and standardize factual, reality-based practices through fictional stories.

¹³ Vogel (1954, 155–156) also mentions that in so-called “real-life problems [transl. by the author]”, not only do the stories seem to be drawn from everyday practice, but they also contain ‘realistic’ values and prices. In contrast, recreational math problems often have unusual settings, and while they also teach valuable mathematical skills, their operations are of a more general nature.

¹⁴ Augsburg, State and City Library, 4° 138, 52v. “Item 54 Man in einem schlos belegert sein ge- / speyset auff 6 monat geburt ydem Zu teglichen / pfrund ein brot 24 lot schwer nach aussgang / dreier monat 12 tag Werden sie aus warnung / erinert sich mit dem ubrigen brot noch 4 monat / 13 tag enthalten müssen wievil lot brots mus / man einer yden personn teglich abbrechen damit / sie angeregte Zeyt hinaus reichen mugen das mo- / nat pro 30 tag Zurayten / Besich wievil korns im schlos sey Nun was / einem davon geburt [...]” Further instructions are provided in the form of individual step-by-step directions for a mathematical problem.

¹⁵ Paris, Bibliothèque Nationale, Lat. 7197, 23v. “Item hette dich ein schöne jungfrow gebetten dz du iren öpfel bre= / chtist uss ein umb gemureten garten da etwe vil tor an wer / und der tor hütter seite wie der gart gefeyet wer dz man im die / öpfel halb sölt geben dar nach dem knecht ouch ein von sinem / teil der die öpfel der jungfrowen oder sus schwanger frowen wil bringen / und als menig tor wer sölt er ouch also tun und dz im eben uber / belib die er verheissen hat uss zebringen nit me noch minder So / merk des ersten wievil der torwarten syen und wie vil du / ietlichem knecht müssist geben und wie vil du her uss müssist / bringen Aber so der torwarten 3 sind und ietlichen knecht 1 gist / und 7 solt her uss bringen So nim und duplier die 7 macht / 14 und tu 2 mal als vil dar zu als du an dem ersten knecht / solt geben macht 16 / Nun duplier die 16 macht 32 [...]”

¹⁶ Examples for this can be found in the collection of medieval recipes that was cataloged as part of the project “CoReMa. Cooking in the Middle Ages” (2026), or, in regard to perspective treatises, in the work of Hieronymus Rodler (cf. Faschingeder / Reisenbichler 2025).

¹⁷ Some examples are so brief that they show no sequence of events, just a description of a situation, for example by providing the exchange rates for a Nuremberg coin and a Venetian coin, and then tasking the recipient with converting a value based on this information. Edith Feistner (2016a, 65) coined the first part of the example ‘narration’, but in this article, based on Michaela Wiesinger (2023, 380), the term ‘exposition’ is used, since it encompasses both narrative and descriptive elements.

¹⁸ Regarding this point, Edith Feistner (2016a, 70) also refers to a break in the narration in favor of mathematical instruction. Feistner explains this break as a shift from the inherently narrative quality of ambiguity to the unambiguity and irrefutability of mathematical processes and results.

¹⁹ Most examples cover a wide variety of traded goods, including pepper, saffron, cloves, cloth, brass, silver, gold, grain and wine, while focusing on exchange rates and different measurements that vary from city to city. Apart from that, other examples focus on different types of commercial calculations, such as partnership accounting, interest calculations, and debt accounting, interwoven with a handful of recreational problems (for instance, testament problems or ring puzzles).

²⁰ See also the work of Hrubý (2021) on ore mining and metal production in the Kingdom of Bohemia and the exhibition catalog on medieval mining in Saxony and Bohemia, edited by Regina Smolnik (2014).

²¹ In his dissertation as part of the ARITHMETIC project, Gregor Kodym examines the dissemination of mathematical knowledge in vernacular literature of the early modern period and the impact of mathematical contents on their stories.

²² Berlin State Library – Prussian Cultural Heritage Foundation, Ms. germ. oct. 375, 1r. “Got hat alle ding be / schaffen zu tzal In gewicht / und In moß Seint Nun yn / allen dingen der werlt dy / tzal ist not zu wissen So / wil ich hie lernen von der / zallichen kunst [...]”

²³ The citation of this biblical verse (Book of Wisdom 11, 20) stems from the groundbreaking arithmetical work of Boethius (*c. 480/485;† between 524 and 526), *De institutione arithmetica*. Boethius’ work, itself an adaption of Greek and Roman mathematics and therefore connecting antique and early medieval mathematics (cf. Herrmann 2024), was a foundation of mathematical instruction in monasteries and universities and a source for many medieval reckoning books in the early Middle Ages. Boethius understood numbers as the fundamental pattern of creation and natural order (Haustein 2004, 105), and therefore considers the number-related subjects of the Quadrivium, founded upon arithmetic, to be the most essential. The biblical verse is also cited in the prefaces or the beginnings of many early modern reckoning books, invoking that God governs all things according to measure, number and weight, and that therefore, teaching the art of numbers is most essential. This reference also emphasizes that the world as whole, beyond the mere economic sphere, can only be understood through mathematical knowledge and thus enhances the relevance of reckoning books.

²⁴ In this context, it is certainly also a testament to the identificatory power of reckoning books that numerous examples feature depictions of stereotypes and ‘othering’, for example in relation to the Jewish population and alleged usury. A part of my dissertation will therefore be concerned with the stereotyping of Jews in reckoning books, e.g. in regard to the mnemonic verse of the ‘Josephsspiel.’ Michaela Wiesinger has also already presented research on findings on the mathematical and social function of the trope of the Jewish usurer. She has discussed this, amongst other instances, in her research paper at the International Medieval Congress (IMC) in Leeds in 2024 under the following title: “Interest or Extortion? The Image of the Moneylending Jew in Reckoning Examples of the Late Middle Ages and the Early Modern Period”, as well as at the conference “Transfer of Ideas in European Intellectual History” held at the University of Innsbruck in July 2025, where she presented the paper: “Usury and Antisemitic Stereotypes: The Extortionist Jew in Reckoning Examples of the Middle Ages and the Early Modern Period.” Norbert Hunor Orbán shows similar stereotyping in Hungarian reckoning books, although in regard to other marginalized groups ‘tailored’ to the specific recipient. His article “Calculating for ‘stubborn Hungarians’ – The ‘Hungarian Frisius’ and its translation” is due to be published in the *Journal of Medieval History*.

²⁵ The name “Enderes Beham” / “Ennderlein Beham” is mentioned on the cover as well as on the first page, fol. 1r.

²⁶ Throughout the text, Behaim keeps track of the continuing dates of the year 1547 in which the parts of the book were written.

²⁷ The addendum regarding the Second Margrave war is found on fol. 96r, the addendum on Behaim's professional trajectory on fol. 96v.

²⁸ Norbert Hunor Orbán (2025) also shows that political conflicts and scenes of crisis are partly reflected in reckoning books.

²⁹ In *Ein lobspruch der statt Nürnberg* (dated to 20th February 1530), Sachs alludes to the city as a "paradeiß" (Keller 1870, 190), a luscious garden inhabited by an eagle sitting on red-white roses (symbolizing the city's coat of arms and its governing city council). The garden and the eagle are constantly surrounded and threatened by other animals, alluding to the emblems of the city's many enemies. Additionally, Nuremberg's population is described as mainly concerned with trade: "Ein grosser thayl treybt kauffmanns-handel" (Keller 1870, 193). See also Meyer (2009, 349).

³⁰ Various variants of the reckoning operation at the core of the apple orchard example and other expositions connected to it can be found in Tropfke (1980, 582-583); Folkerts (1971, 69–71) and Singmaster (2004).

³¹ A similar icon and Latin epigram can already be found in earlier editions of the emblem book, for instance in the first edition of 1531 and the extended edition of 1534.

³² "Der apfel ein hitzige Frucht; Damit Venus oft hat geband zwen gmahel." Alciato, Andrea: *Clarissimi viri D. Andreae Alciati Emblematum libellus*. Munich, Bavarian State Library, Res / L.eleg.m. 35, pp. 138–139.

³³ Of course, as Tomasek (1994, 12–20) states, not every riddle has to be solvable: Solvability and unsolvability are more of a spectrum along which riddles fall to varying degrees. For a reckoning example, solvability is a core element of its didactic effect and is therefore essential.